vibration and gives information on the relative importance of various elements to system stability.

To eliminate the vibration, hydraulic accumulators were placed in the propellant suction lines near the pump inlets. This is shown schematically in the left fuel line in Fig. 2. The accumulator is a spring mass device that alters the dynamic characteristics of the suction line. By proper choice of accumulator constants, the propulsion system can effectively be decoupled from the structure. Mathematically, the accumulator is included in the model through the equation

$$(L_aS^2 + C_aS + B)W_a = -P_s \tag{10}$$

and the addition of the last term in Eq. (3). To date, accumulators have been incorporated in four Titan II flights and have essentially eliminated the vibration phenomenon.

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Ballistics of Solid Propellants during Thrust Modulation

B. Dubrow,* E. D. Guth,† and M. W. Wong†
TRW Space Technology Laboratories, Redondo Beach, Calif.

Nomenclature

- a = reaction constant from the Summerfield burning rate relationship
- A_b = area of propellant burning
- A_t = area of the throat, thrust modulating nozzle
- A_{t_0} = area of the throat, initial
- b = diffusion constant from the Summerfield burning rate relationship
- C_W = mass flow factor
- M = molecular weight
- P = pressure
- P_0 = initial pressure

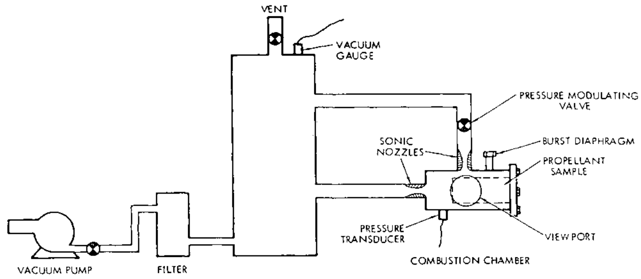

Fig. 1 Schematic diagram of experimental apparatus.

- R = gas constant
- r = linear burning rate of solid propellant
- T = absolute temperature
- t = time required for the pressure to change from P_0 to P
- V = volume
- ρ = propellant density
- ρ_g = gas density

Introduction

THRUST modulation of conventional solid propellant rockets can be made to occur through controlled pressure excursions in the rocket chamber. The chamber pressure, in turn, can be varied by changes in the propellant burning surface area, the propellant properties, or the nozzle throat area. Changing the nozzle throat area is considered as the method that lends itself most readily to reducing the concept of thrust modulation to practice. In order to determine the feasibility, problems, and design criteria related to thrust modulation of solid propellants by programed variations of the nozzle throat area, the solid propellant ballistics must be characterized under transient conditions of pressure rise and decay.

The reported experimental investigations of pressure decay did not appear to be devised for general analytical treatment. Furthermore, the efforts were aimed at generating data under the limiting conditions in the region of combustion extinction. The analysis of programed thrust modulation requires more inclusive experimental conditions. In the interest of arriving at a general treatment of thrust modulation, an experimental program was devised which could generate data required for analytical assessment of the related ballistics.

Analytical Treatment of Transient Ballistics

The internal ballistics for a solid rocket can be expressed analytically in general form by the mass balance equation:

$$Vd\rho_g/dt = \rho A_b r - C_W P A_t \tag{1}$$

If the propellant continues to burn during thrust modulation, introduction of an expression of the burning rate as a function of pressure permits Eq. (1) to be solved for time as a function of pressure. The Summerfield equation for pressure-dependence of burning rate is a two-term parametric formula:

$$1/r = a/P + b/P^{1/3} \tag{2}$$

Equation (2) considers the combustion mode for solid propellants as consisting of two rate-controlling phenomena. At low pressure, chemical reaction is rate controlling, whereas diffusion is the predominant rate controlling process at high pressure.

Equation (1) can be solved when Eq. (2) is used to describe the burning rate, yielding

$$t = \frac{MV}{RT}\left\{\frac{a}{A_b\rho - aC_W(A_{t_0} + A_t)}\ln\frac{P}{P_0} + \frac{3}{2}\left[\frac{C_W(A_{t_0} + A_t)}{A_b\rho - aC_W(A_{t_0} + A_t)}\right] \times \ln\left[\frac{[A_b\rho/C_W(A_{t_0} + A_t)] - (a + bP^{2/3})}{[A_b\rho/C_W(A_{t_0} + A_t)] - (a + bP^{2/3})}\right]\right\} \tag{3}$$

Presented as Preprint 64-130 at the AIAA Solid Propellant Rocket Conference, Palo Alto, Calif., January 29–31, 1964; revision received November 2, 1964.

* Member of the Technical Staff. Member AIAA.

† Member of the Technical Staff.

Table 1 Transient and steady-state ballistic parameters

Propellant type[a]	A_1	A_2	A_3	B_1	B_2	C_1	C_2
Summerfield constants calculated from:							
1) Transient pressure decay							
a	400	400	350	400	490	400	425
b	11	9	7	4.5	10	4	10
Calculated "burning rate" at 1000 psia, in./sec	0.667	0.769	0.633	1.180	0.671	1.250	0.702
2) Transient pressure rise							
a	400	900	665	...	950	...	640
b	5	5.6	5.1	...	2.7	...	7.9
Calculated "burning rate" at 1000 psia, in./sec	1.111	0.683	0.850	...	0.820	...	0.751
3) Steady-state burning							
a	242	...	257	...	233	217	...
b	27	...	19.5	...	19	9.2	...
Calculated burning rate at 1000 psia, in./sec	0.339	...	0.452	...	0.468	0.878	...

Code: [a] Polyurethane, A; Polybutadiene, B; double base, C;

Implied in the derivation and application of Eq. (3) as well as Eq. (2) are the following underlying assumptions: 1) the pressure can be considered constant throughout the chamber, 2) the combustion temperature and mass flow factor are approximately constant, 3) the chamber free volume and propellant burning surface are approximately constant, 4) the molecular weight of the combustion products does not change, and 5) sonic flow exists at all times.

Thus, for a pressure excursion in a rocket motor from P_0 to P, caused by a rapid change in nozzle throat area from A_{t0} to $A_{t0} + A_t$, Eq. (3) should provide a complete description of the transient ballistic event provided that the Summerfield physicochemical model is valid under these conditions.

Experimental Approach

In the experimental equipment, which is shown schematically in Fig. 1, right cylindrical propellant samples, 3 in. in diameter and up to 5 in. in length, were burned.

The gases exited through the main nozzle. The pressure modulating valve was opened or closed on signal, thus producing programed fluctuations in the chamber pressure. Output data for a typical two cycle pressure modulation are shown in Fig. 2.

Discussion

The Summerfield constants, a and b, predominating during low- and high-pressure steady-state combustion, respectively, were obtained by iterative fit of the experimental pressure-time curves. The typical fit of Eq. (3) to the experimental data is illustrated in Fig. 3. Table 1 presents the best-fit transient constants for the propellants studied. Repetitive tests of each propellant type appeared to be characterized by specific transient constants. The steady-state values of the Summerfield constants also are given for comparison in Table 1.

The Summerfield constant a obtained from the transient data is consistently higher than the steady-state value. The change in apparent burning rate with pressure, therefore, is higher. This indicates that the chemical kinetic mechanism is more predominant during transient conditions. The chemical kinetic constant for pressure decay was found to be approximately 400 for all propellants tested. The variation from this value is within the error of determining best fit. Such a consistent trend in the data for each propellant was not obtained for the pressure-rise transients. It was also noted, however, that the pressure-rise phenomenon was less reproducible.

Samples of polyurethane and polybutadiene propellants were extinguished during pressure decay. When considering this event, it is important to recognize that propellant extinction was not the intended purpose of the experiment. Therefore, discussion of relative extinction parameters does not appear warranted. The test results, however, clearly substantiate that propellant combustion can be extinguished through pressure decay.

When interpreting the experimental data, motor design parameters such as dimensions, grain configuration, and nozzle orientation must be considered. In general, the characterization of transient ballistics which is developed in this paper provides an analytical description of pressure ex-

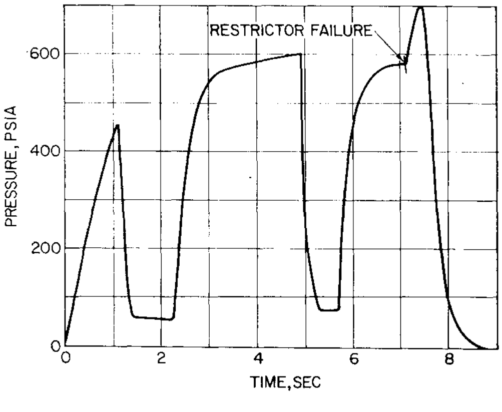

Fig. 2 Two-cycle pressure modulation.

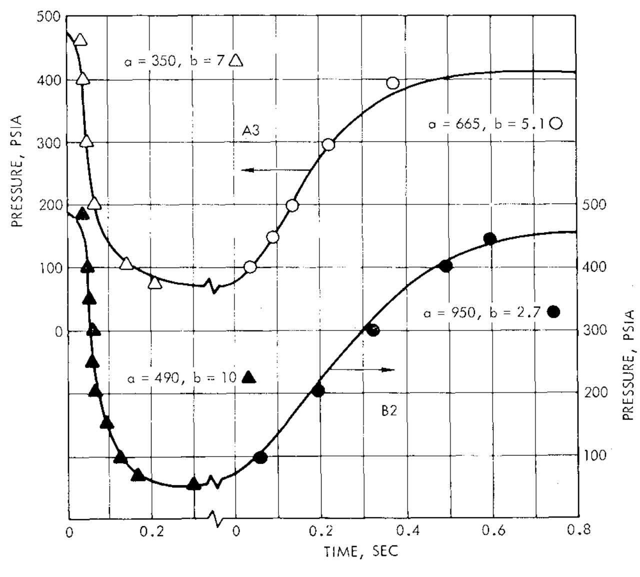

Fig. 3 One-cycle modulation of propellants.

cursions produced by an end-burning configuration and caused by rapid changes in exhaust area. The specific application of the experimentally derived parameters to other configurations must be tested.

Conclusions

The experimental results show that solid propellants can be thrust modulated over a wide range, and extinction of combustion can occur at the limiting conditions of pressure decay. The pressure excursions during thrust modulations are amenable to analytical treatment. The characterization of repetitive pressure decay and subsequent rise by the combined analytical and experimental methods provdies valuable insight in designing a solid propellant rocket motor capable of thrust modulation.

Atmospheric Acoustics as a Factor in Saturn Static Testing

Richard N. Tedrick*
NASA George C. Marshall Space Flight Center, Huntsville, Ala.

ONE facet of the static test firing of large space vehicles has been the generation of large amounts of acoustic energy. The rapid increase in the size of these boosters during the last few years and the resultant increase in the noise levels generated during their static testing has made the prediction and control of sound generation an important phase of rocketry. This has been true especially since the advent of the Saturn vehicles, which are not only the world's largest tools for extraterrestial investigation but are also the most powerful manmade, steady-state noise generators. Results from field surveys of the noise show that the acoustic power radiated amounts to about $\frac{1}{2}$ of 1% of the total mechanical power of the engines. In the case of the Saturn S-I, this means about 40 Mw. Because of the meteorological factors at the time of firing, this acoustical energy has sometimes been concentrated into relatively small zones in business or residential areas. Such occurrences have heightened the interest in determining what may be the acoustic consequences of static firing even larger rocket vehicles.

Generally, it may be said that the larger the space vehicle that is being tested, the larger is the amount of sound which is radiated into the atmosphere. However, there are two additional factors that greatly affect the response that may be anticipated from the surrounding communities. One of these is the frequency content of the sound from the engine test. It has been shown[1] that, as the thrust of the rocket engine goes up, the peak frequency goes down. This affects the sound level at long ranges because the lower frequencies (below 100 cps) do not attenuate as rapidly. Thus, a larger percentage of the original sonic energy is left to disturb outlying areas. Also, as the peak drops in frequency, additional energy is put into the subaudible range. Since it is these lower frequencies that rattle windows and shake buildings, the "alarm level" is expected to rise with larger boosters.

Another factor affecting the amount of acoustic energy which reaches the surrounding areas is what is known as the "directivity" of the source. This is simply an index of the relative amounts of energy which are directed by the source itself in each direction. Contributing to this are not only the rocket engine and exhaust velocity parameters but also the shape and configuration of the flame deflector and test tower.

Presented as Preprint 64-207 at the 1st AIAA Annual Meeting, Washington, D. C., June 29–July 2, 1964; revision received August 17, 1964.

* Acoustic Physicist, Test Laboratory. Member AIAA.

After the sound has been radiated into the atmosphere, several things can happen: 1) the sound can be propagated normally, as in a still room or large stadium where the effects of wind and temperature are negligible (as on a very still and quiet morning); 2) it can be directed into the upper atmosphere to be dissipated; and 3) it can be directed toward one or more locations on the earth's surface.

To avoid the acoustic problems inherent in the static testing of large space vehicles, a program of "selective firings" has been instituted at Marshall Space Flight Center (MSFC). Methods for forecasting and evaluating the undesirable firing conditions and for locating the areas that may be adversely affected by returning sound have been developed. These have been based upon acoustic and atmospheric soundings for the 36-hr period immediately preceding such a test. This program not only protects the surrounding communities but also allows maximum scheduling flexibility to the test engineer.

Description of the Problem

The Saturn S-I radiates its 40 Mw of power into an atmospheric hemisphere with a very broad directivity[2] and continues over an operational period of approximately 2 min. Since much of its energy is well below 100 cps, the resonances of local structures are sometimes reached.

Under certain unfavorable atmospheric conditions, those sound rays emanating from the source at angles with the horizontal up to 20° or more can be refracted such that they return to the earth's surface at considerable distances and focus a seriously high acoustic intensity within a relatively small area. Thus, on occasion, propagated sound from the Saturn test has produced annoyance and alarm at ranges of 10 miles or more within the city and suburbs of Huntsville.

Actual sound fields that exist in typical out-of-doors situations are almost prohibitively difficult to describe in detail. Since the medium for acoustic transmission is the atmosphere, it is never either homogeneous or quiescent and the boundary conditions are often quite complicated in terms of contour, vegetative covering, and manmade structures. However, it is possible to treat the problems in an approximate way by considering the following principal elements of sound propagation theory: 1) attenuation by spherical divergence, or the spreading out of the wave front; 2) attenuation due to the mechanical properties of the molecular structure of the atmosphere; 3) attenuation due to ground effects along the earth's surface; and 4) attenuation by refraction of sound fronts resulting from spatial variations in air temperature and wind. The most important of the foregoing elements, in terms of the Saturn noise problem, is the refraction effect, which is responsible not only for bending the sound rays back to the earth but often results in focal areas of concentrated sound energy.[3]

Calculation of Refractive Effect

Since the sound velocity in air depends upon temperature, humidity, and wind, it is the variation of these factors with altitude which determines the verticle sound velocity gradient and ultimately the refraction of sound waves. Considering first the effects of temperature and humidity, the sound speed C in still, dry air is given by LaPlace's equation

$$C = K\,(T^*)^{1/2} \tag{1}$$

where K is a constant (20.07 for C in meters per second) and T^* is in degrees Kelvin. The virtual temperature is T^*, which is defined as that temperature for the density of a parcel of dry air to equal that of moist air under the same pressure. Virtual temperature is related to the actual temperature T by the following expression:

$$T^* = T/[1 - 0.377\,(e/p)] \tag{2}$$

where e is the water vapor pressure and p the total pressure.